babelbabel2008/03/16stringencnameref[2012/07/28]
seminar

A Simple Planning Problem for COVID-19 Lockdown

Fernando Alvarez — University of Chicago
David Argente — Pennsylvania State
Francesco Lippi — LUISS and EIEF

ANCE – Virtual Seminar

May 15, 2020

Overview

- Adopt (S, I, R) dynamic epi model: $S(t) \rightarrow I(t) \rightarrow R(t)$
- Contagion through social interactions implies externalities
- Planner's Problem: trade-off PV economic activity vs "value of life"

Overview

- Adopt (S, I, R) dynamic epi model: $S(t) \rightarrow I(t) \rightarrow R(t)$
- Contagion through social interactions implies externalities
- Planner's Problem: trade-off PV economic activity vs "value of life"
- Model Key Features:

– (1) **Congestion** of health-care affects fatality rate: $\phi(I)$

– (2) **Lockdown** affects infected (I) and susceptible (S) and....:
  - – ....**with Antibody Test**: R can be identified & thus not in lockdown
  - – ....**w/o Antibody Test**: R must be in lockdown.

– (3) Analyze extension with **Test-Trace-Quarantine** (TTQ)

Overview

- Adopt (S, I, R) dynamic epi model: $S(t) \to I(t) \to R(t)$
- Contagion through social interactions implies externalities
- Planner's Problem: trade-off PV economic activity vs "value of life"
- Model Key Features:
  - (1) **Congestion** of health-care affects fatality rate: $\phi(I)$
  - (2) **Lockdown** affects infected (I) and susceptible (S) and....:
    - ....**with Antibody Test**: R can be identified & thus not in lockdown
    - ....**w/o Antibody Test**: R must be in lockdown.
  - (3) Analyze extension with **Test-Trace-Quarantine** (TTQ)
- Results:
  - Identify **main determinants for lockdown intensity and duration**
  - Quantify **welfare costs of lockdown** (forgone GDP and lost lives)
  - Quantify benefits of **Testing** and of **TTQ**

Related econ. literature

Many recent studies, some closely related, most date March / April 2020

- Planning problem: Hansen-Troy 2011; Eichenbaum,Rebelo and Trabandtz; Gonzalez-Eiras and Niepelt; Kaplan, Moll and Violante; Piguillem and Shi
- Equilibria with (endogenous) social distancing: Fenichel (2013); Toxvaerd (2015); Jones, Philippon and Venkateswaran ; Farboodi,Jarosch,Shimer
- forecasting, measurement and cost valuations : Atkeson; Bassetto; Hall,Jones,Klenow; Stock; Jones-Villaverde

Setup: SIR model with Lockdown Policy

- Total Population at t: Susceptible + Infected + "Recovered"

$$N(t) = S(t) + I(t) + R(t)$$

- Lockdown fraction $L(t) \in [0, \bar{L}]$ of $S(t) + I(t)$,

- Susceptible become infected by contacting I at rate: β

$$\dot{S}(t) = -\beta \quad S(t) \times I(t)$$

- Infected transit to "recovered" at rate γ

$$\dot{I}(t) = \beta \quad S(t) \times I(t) \qquad - \gamma \; I(t)$$

-

Setup: SIR model with Lockdown Policy

- Total Population at t: Susceptible + Infected + "Recovered"

$$N(t) = S(t) + I(t) + R(t)$$

- Lockdown fraction $L(t) \in [0, \bar{L}]$ of $S(t) + I(t)$, with effectiveness $\theta \in (0, 1)$

- Susceptible become infected by contacting I at rate: $\beta(1 - \theta L(t))^2$

$$\dot{S}(t) = -\beta \quad S(t) \times I(t)(1 - \theta L(t))^2$$

- Infected transit to "recovered" at rate γ

$$\dot{I}(t) = \beta \quad S(t) \times I(t)(1 - \theta L(t))^2 - \gamma \; I(t)$$

-

Setup: SIR model with Lockdown Policy

- Total Population at t: Susceptible + Infected + "Recovered"

$$N(t) = S(t) + I(t) + R(t)$$

- Lockdown fraction $L(t) \in [0, \bar{L}]$ of $S(t) + I(t)$, with effectiveness $\theta \in (0, 1)$
- Susceptible become infected by contacting I at rate: $\beta(1 - \theta L(t))^2$

$$\dot{S}(t) = -\beta \quad S(t) \times I(t)(1 - \theta L(t))^2$$

- Infected transit to "recovered" at rate γ

$$\dot{I}(t) = \beta \quad S(t) \times I(t)(1 - \theta L(t))^2 - \gamma \; I(t)$$

- A fraction of the infected die

$$-\dot{N}(t) = \underbrace{[\varphi + \kappa \; I(t)]\,\gamma}_{\phi(I)} \times I(t)$$

Planner's dynamic control problem

- Planner chooses path $\{L(t)\}$ to minimize the present discounted value:

$$V(S,I) = \min_{L(t)} \int_0^\infty e^{-(r+\nu)t} \left(\underbrace{w\, L_t\, [\tau(S_t + I_t) + 1 - \tau]}_{\text{GDP loss}(t)} + \underbrace{\phi(I_t)\, I_t \times [vsl]}_{\text{deaths}(t)\times\text{value of life}} \right) dt$$

subject to the law of motion of S and I shown before.

- Two technologies for lockdown

– w. Test ($\tau = 1$) : lockdown to ($S + I$), not to R

– w/o Test ($\tau = 0$): lockdown to ($S + I + R$)

Bellman equation

Value function $V(S, I)$ for all $0 \leq S + I \leq 1$ solves:

$$(r+\nu)V(S,I) = \min_{L\in[0,\bar{L}]} w\, L\, [\tau(S+I) + 1 - \tau] + \phi(I)\, I\, [\text{vsl}] \\ - \left[\beta\, SI\, (1-\theta L)^2\right]\partial_S V(S,I) + \left[\beta\, SI\, (1-\theta L)^2 - \gamma I\right]\partial_I V(S,I)$$

Initial condition $N(0) = 1$, $I(0) = \epsilon$ and $S(0) = 1 - \epsilon$.

Value function domain $(S, I) \in (0, 1) \times (0, 1)$ s.t. $S + I < 1$

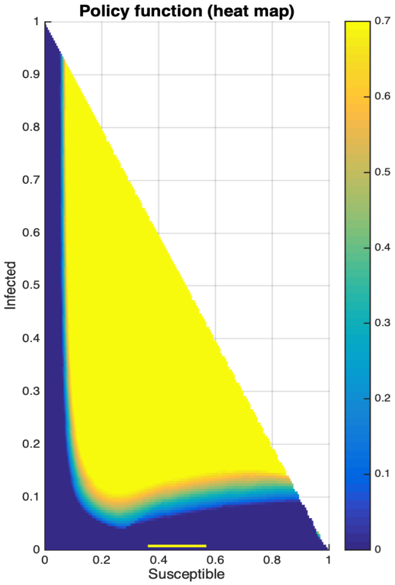
Boundary condition at $I = 0, \forall S \in (0, 1)$: $V(S, 0) = 0$

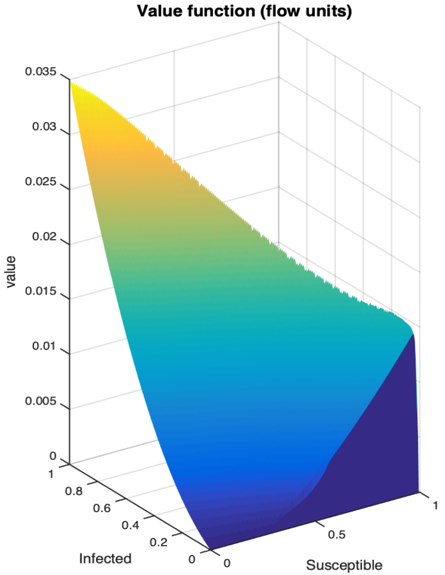
Boundary condition at $S = 0, \forall I \in (0, 1)$: $V(0, I) = vsl \cdot \left(\frac{\varphi\gamma}{r+\nu+\gamma} + \frac{\kappa\gamma I}{r+\nu+2\gamma}\right) \cdot I$

Baseline parameters

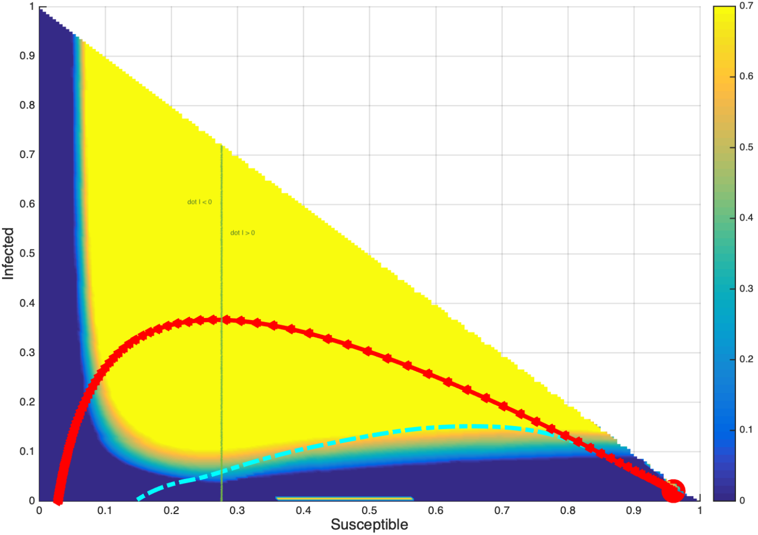
Parameter	Value	Definition
		EPI PARAMETERS
β	$1/5 \times 365$	Annual Transmission rate if (uncontrolled)
γ	$1/18 \times 365$	Annual rate at which active cases "recover"
φ	0.01	Deaths per active case (per day) at zero infections.
κ	0.05	Implies a 3 % fatality rate with 40 % infected.
		OTHER PARAMETERS
r	0.05	Annual interest rate 5 percent.
ν	0.667	Prob rate vaccine + cure (exp. duration 1.5 years)
$\bar{L}$	0.70	1 - GPD share health, retail, gov., utilities, food mfg.
θ	0.5	Lockdown Effectiveness (0 is ineffective)
vsl	20	Value of Statistical Life = 20 $\times$ GPD per capita (Jones, Hall and Klenow)

Value Function and Optimal Policy

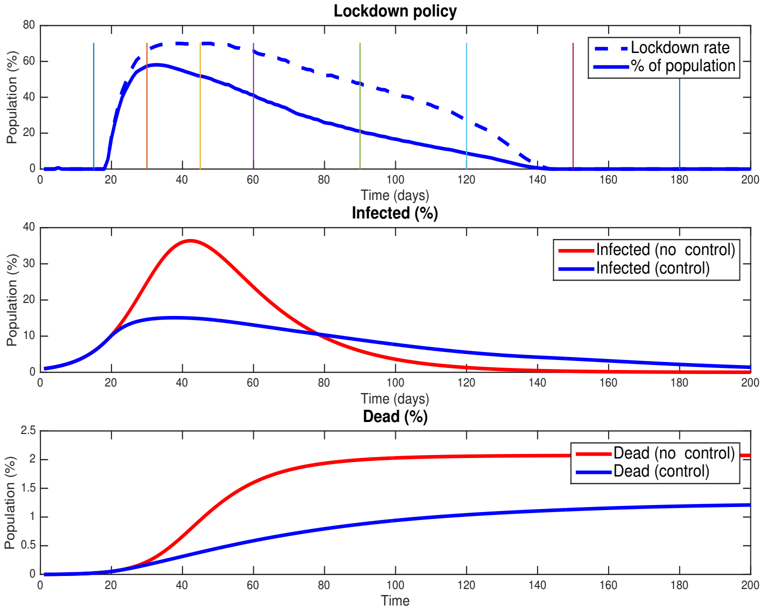




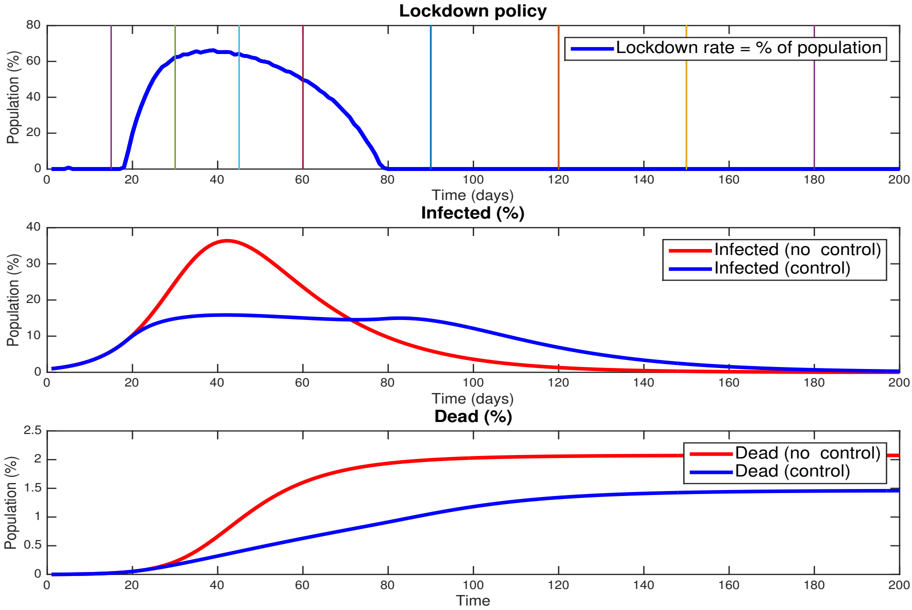
Optimal Policy, phase diagram (bench., $I(0) = 0.01$)



Time path: Benchmark w. Testing ($\tau = 1$)

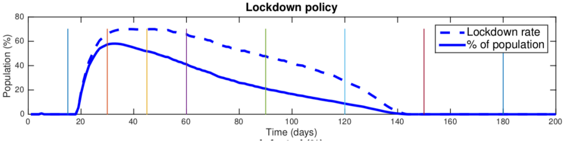


Time path: Benchmark w/o Testing ($\tau = 0$)

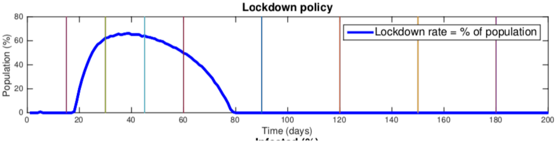


Welfare effect of Testing: 2% of one year's GDP

Panel A – Case w / testing ($\tau = 1$)



Panel B – Case w/o testing ($\tau = 0$)



Remarks on the optimal lockdown paths:

-1- Lckdn. much longer with testing, and ends gradually

-2- Lckdn. ends abruptly w/o testing (large efficiency costs)

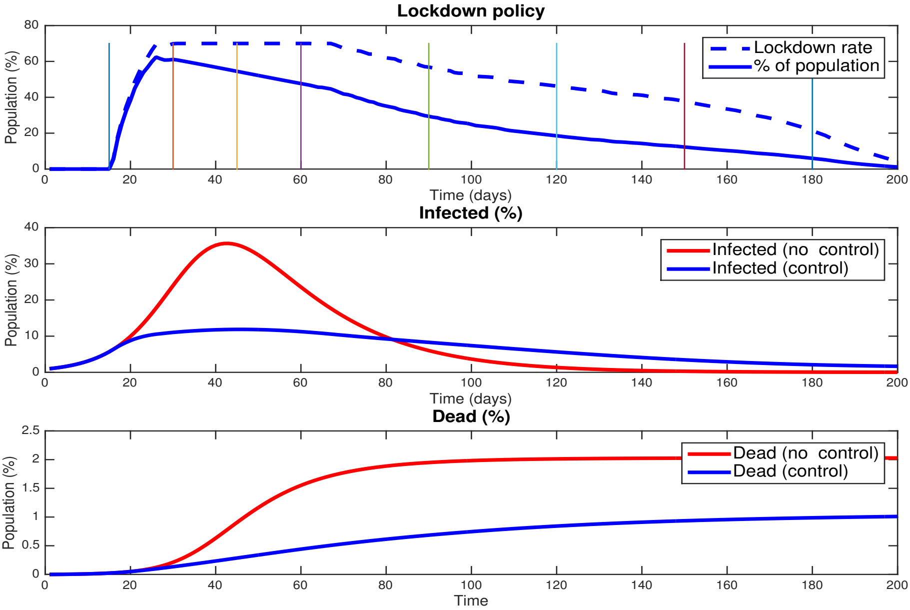
-3- Lckdn.s comparable as total hours lost; but deaths higher w/o Test

Welf. Loss w. Test: Optimal policy vs. No Intervention

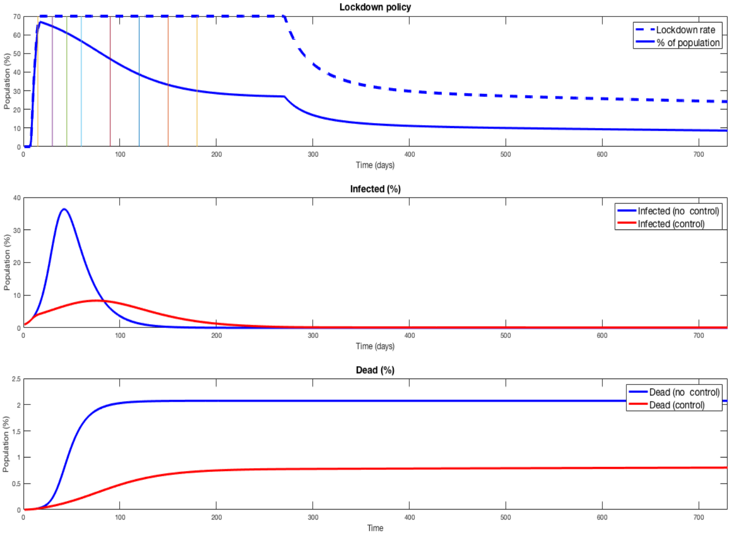
Case	Optimal Policy Welfare Loss	Optimal Policy Output Loss	No Policy Welfare Loss
Benchmark Case			
Medium lock. effectiveness (θ=0.5)	1.5%	0.4%	1.9%
High lock. effectiveness (θ=0.7)	1.4%	0.5%	1.9%
Alternative Values of Statistical Life (baseline is 20$\times$ GDP/capita)			
vsl = 30$\times$ GDP/capita	2.0 %	0.6 %	2.8 %
vsl = 80$\times$ GDP/capita	3.7 %	1.4 %	7.5 %
Constant fatality rate (κ=0)			
Medium lock. effectiveness (θ=0.5)	0.9 %	0.0 %	0.9 %
High lock. effectiveness (θ=0.7)	0.9 %	0.0 %	0.9 %

Multiply %'s by 20 to get cost as a fraction of one year's GDP

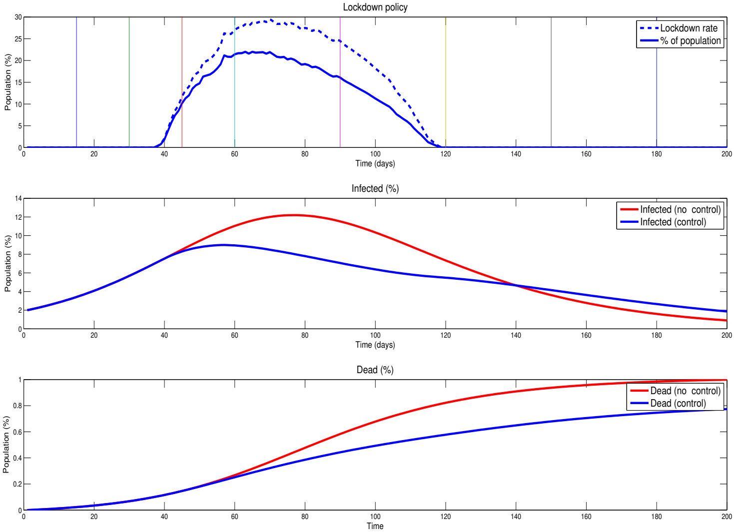
Time path, higher VSL ($vsl = 30\times$ GDP/capita)



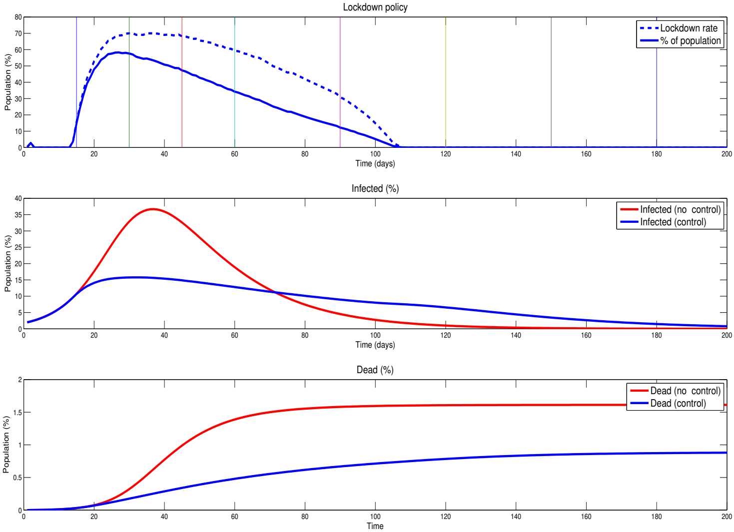
Time path: very high VSL ($vsl = 60\times$ GDP/capita)



Time path: Lower Transmission Rate ($\beta = 0.10$)



Time path: Lower Fatality Per Active Case ($\varphi = 0.005$)



More scenarios: NO test and lower lethality of virus

Case	Optimal Policy		No Policy
	Welfare Loss	Output Loss	Welfare Loss
Benchmark Case w. Test			
vsl $= 20$; $\beta = 0.20$; $\tau = 1$; $\varphi = 0.01$	1.5 %	0.4 %	1.9%
Benchmark w/o Test			
	1.6 %	0.4 %	1.9 %
Lower diffusion / Fatality rates			
Slower diffusion ($\beta = 0.20/2$)	0.8%	0.1%	0.8%
Lower fatality rates ($\varphi = 0.01/2$)	1.1%	0.4%	1.5%

Multiply %'s by 20 to get cost as a fraction of one year's GDP

Extension: Testing, Tracing and Quarantine (TTQ)

New control T: people sent to quarantine (flow);

$$\begin{aligned}\dot{S}_t &= -\beta S_t(I_t - Q_t)(1-\theta L_t)^2 \\ \dot{I}_t &= \beta S_t(I_t - Q_t)(1-\theta L_t)^2 - \gamma I_t \\ \dot{Q}_t &= T_t - \gamma Q_t\end{aligned}$$

The planner minimizes (Assume $\tau = 1, \kappa = 0$):

$$V(S_0, I_0, Q_0) = \min_{\{L_t, T_t\}} \int_0^\infty e^{-(r+\nu)t} \Big\{ wL_t \Big[(S_t + I_t - Q_t)\Big] + wQ_t + \\ + c(T_t; S_i, I_t - Q_t) + vsl\,\varphi\gamma I_t \Big\} dt$$

Direct cost of test-tracing T people:

$$c(T, S, I-Q) = \frac{\alpha}{2}\left(T \underbrace{\left(\frac{S+I-Q}{I-Q}\right)^{1-\zeta}}_{\#\text{``task'' per } T} \right)^2$$

Effectiveness of tracing: $\zeta \in [0, 1]$;

Extension: Testing, Tracing and Quarantine (TTQ)

New control T: people sent to quarantine (flow); Define $X_t \equiv I_t - Q_t$

$$\dot{S}_t = -\beta S_t(I_t - Q_t)(1 - \theta L_t)^2$$
$$\dot{I}_t = \beta S_t(I_t - Q_t)(1 - \theta L_t)^2 - \gamma I_t$$
$$\dot{Q}_t = T_t - \gamma Q_t$$

The planner minimizes (Assume $\tau = 1, \kappa = 0$):

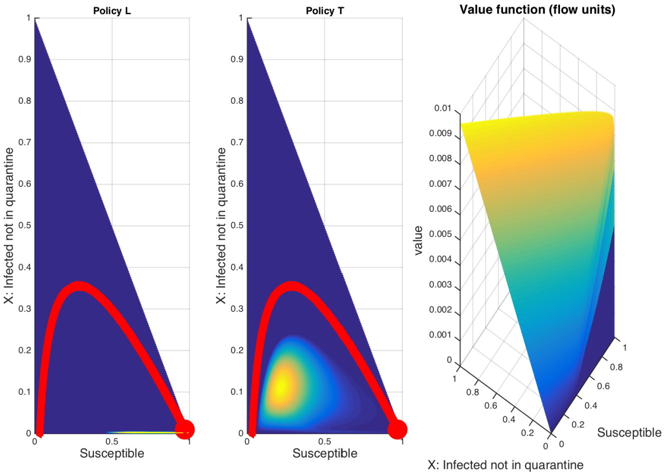
$$V(S_0, I_0, Q_0) = \min_{\{L_t, T_t\}} \int_0^\infty e^{-(r+\nu)t} \Big\{ wL_t \Big[(S_t + I_t - Q_t) \Big] + wQ_t + \\ + c(T_t; S_i, I_t - Q_t) + vsl\, \varphi\gamma I_t \Big\} dt$$

Direct cost of test-tracing T people:

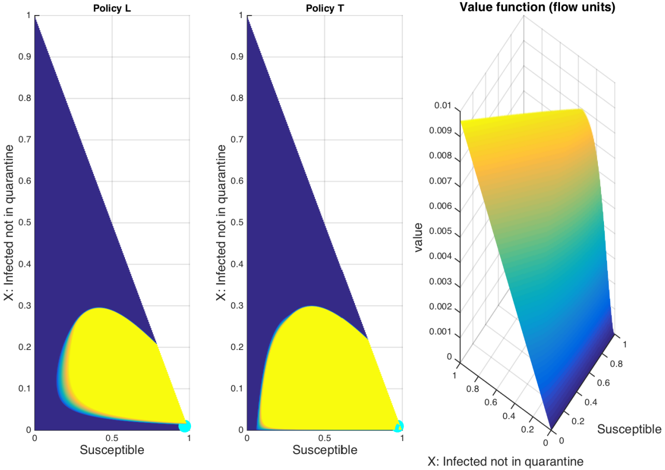
$$c(T, S, I - Q) = \frac{\alpha}{2} \left(T \underbrace{\left(\frac{S + I - Q}{I - Q} \right)^{1-\zeta}}_{\#\text{“task” per } T} \right)^2$$

Effectiveness of tracing: $\zeta \in [0, 1]$; Prop: $V(S, I, Q) = v(S, X) + Q\frac{w + vsl\varphi\gamma}{r + \nu + \gamma}$

TTQ - Random Tracing ($\zeta = 0$)

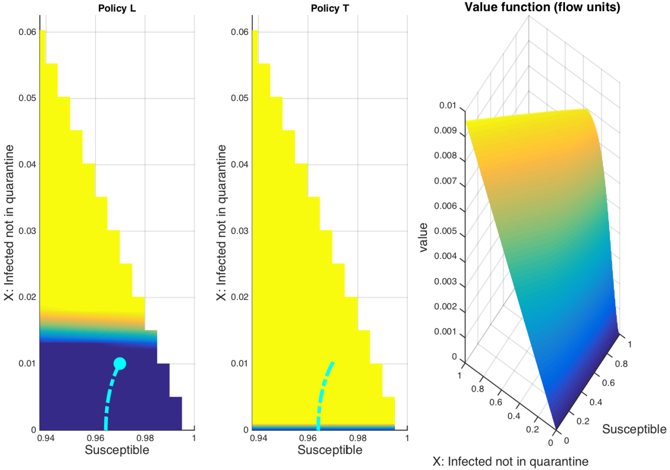


TTQ - Perfect Tracing ($\zeta = 1$)



Perfect tracing reduces welfare losses by 4 times (from 0.9% to 0.2% GDP)

TTQ - Perfect Tracing ($\zeta = 1$) – Zoom on details



Perfect tracing reduces welfare losses by 4 times (from 0.9% to 0.2% GDP)

Conclusions

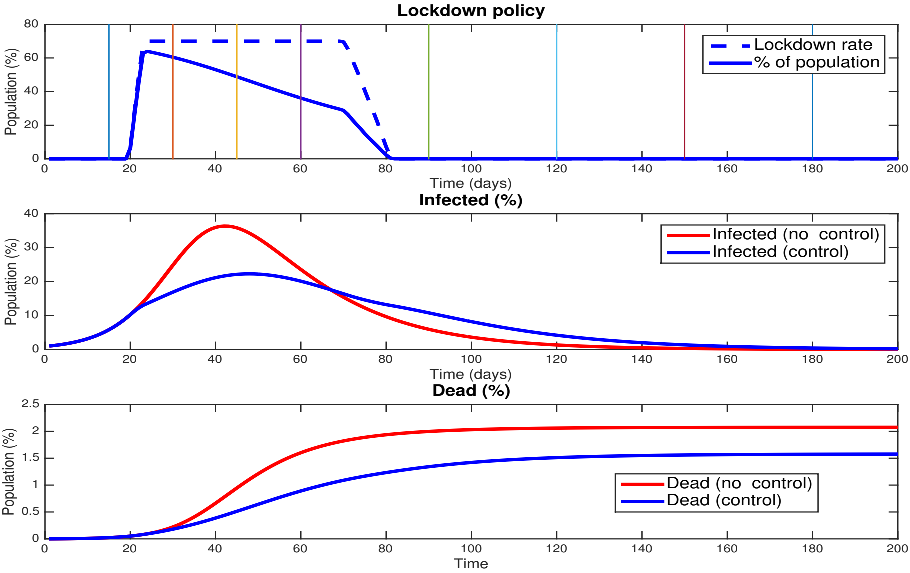
- Simple framework to analyze trade off of lockdown
- COVID19 is very costly: PV of forgone output is approx 8% annual GDP
- The benefit of a test for the recovered is large (2% GDP)
- TTQ policy can be valuable with effective tracing

Conclusions

- Simple framework to analyze trade off of lockdown
- COVID19 is very costly: PV of forgone output is approx 8% annual GDP
- The benefit of a test for the recovered is large (2% GDP)
- TTQ policy can be valuable with effective tracing
- Main forces behind optimal lockdown intensity and duration
  - gradient of fatality rate $\phi(I)$
  - statistical value of life
- Open questions
  - Parameters? hard to measure, fatality rates, β, congestion, ...
  - Richer model: Non linear cost of lockdown, persistent effects
  - Equilibrium: self-isolation, alternative matching framework
- Even w/small state space, tricky computation at the border of state space.

Additional slides

Time path: Low effectiveness ($\theta = 0.3$ vs $\theta = 0.5$)



Time path: Lower VSL ($vsl = 10\times$ ann. GDP/capita)